Dynamical systems in high dimension: methods and applications.

This series of lectures aims at reviewing the research activity on the study of dynamical systems in high dimension. This subject appears in several contexts: in physics, understanding the many body dynamics of complex systems is essential to characterize their equilibration properties and/or lack thereof. Beyond physics, understanding the dynamics of optimization algorithms is essential when the corresponding optimization problems are high-dimensional and non-convex. For example this is the typical case of the training dynamics of artificial neural networks. The purpose of this course is to review what we know about dynamical systems in high dimension in several contexts and to discuss dynamical mean field theory which is the main toolbox to study these problems.

Outline:
1) Stochastic differential equations, Langevin dynamics, Fokker-Planck equation and stationary measures. Equilibrium dynamics and Fluctuation-Dissipation relations.
2) Dynamical mean field theory: Dynamical cavity method
3) Dynamics of mean field spin glasses. High temperature phase and relaxation to equilibrium; Low temperature phase, separation of timescales and aging solution. Landscape interpretation. Quasi-equilibrium dynamics. Breakdown of the aging solution. Universality, marginal stability and stochastic stability out-of-equilibrium.
4) High-dimensional inference: Langevin/Gradient Descent algorithms and their suboptimality with respect to Approximate Message Passing.
5) Dynamical mean field theory: supersymmetric formalism
6) Continuous constraint satisfaction problems and artificial neural networks. Gradient Descent, Stochastic Gradient Descent and Online learning.
7) Biological (Recurrent) neural networks. Transition to and chaos in high dimension. Maximal Lypaunov exponent. Learning algorithms and optimal control of chaotic dynamics.
8) Dynamics of generative diffusion.
9) Open problems and perspectives